Problem:

Solve the problem, using the majorant method:

$\log_{\frac{1}{2}}(x+1)+\cos^{-1}(x+y^2)\le -1.$

Solution:

From the definitions of logarithm and arccos $\Rightarrow \begin{cases} x+1>0 \\ -1\le x+y^2\le 1 \end{cases}.$

We also have the range of arccos values:

$-[0;\pi],\Rightarrow\ \cos^{-1}(x+y^2)\ge \log_{\frac{1}{2}}(x+1)\Rightarrow \log_{\frac{1}{2}}(x+1)\le -1,\qquad \frac{1}{2}<1\Rightarrow x+1\ge\left(\frac{1}{2}\right)^{-1}=2$

(the exponential function a^x is decreasing, when $a<1$), $\Rightarrow x\ge 10.$ On the other hand $1\ge x+y^2\ge 1+$ $+y^2\Rightarrow y^2\le 0\Rightarrow y=0, 1\le x\le 1\Rightarrow x=1,\Rightarrow$

$\Rightarrow \log_{\frac{1}{2}}(x+1)+\cos^{-1}(x+y^2)=\log_{\frac{1}{2}}2+\cos^{-1}1=-1+0=-1.$

We got the unique solution: $x=1,\ \ y=0.$

Answer: $x=1,\ \ y=0.$