Problem:

Solve the system of equations:

$$\begin{cases} x-2y+\dfrac{3}{x-2y}=4 \\ \dfrac{y}{x-2y-3}=5 \end{cases}$$

Solution:

$$\begin{cases} x-2y+\dfrac{3}{x-2y}=4 \\ \dfrac{y}{x-2y-3}=5 \end{cases} \Rightarrow \begin{cases} (x-2y)^2-4(x-2y)+3=0 \\ y=5(x-2y-3) \end{cases} \quad (*)$$

We see from the first equation that this is a quadratic equation for $(x-2y) \Rightarrow x-2y=2\pm\sqrt{4-3}=2\pm$ $\pm 1 \Rightarrow$ let's consider the cases:

1) $x-2y=2-1=1, \Rightarrow$ from the 2nd equation $(*) \Rightarrow y=5(x-2y-3)=5(1-3)=-10, \Rightarrow x=2y+$ $+1=-19$.
2) $x-2y=2+1=3, but\ then\ x-2y-3=0$, which is impossible (in the initial system $x-2y-3\neq 0$).

Answer: $x=-19, y=-10$.