Problem:

Find all values of the parameter a, for which

1) the equation $ax^2 + x + 17 = 0$ has only one root;
2) the equation $ax^2 + (5 - 3a)x - a = 0$ has two roots of different signs.

Solution:

1) If $a = 0 \Rightarrow x = -17$ is the only root. If $a \neq 0$, both roots of the equation are equal $\Rightarrow$

$$D = 1 - 4 \cdot 17 \cdot a = 0 \Rightarrow a = \frac{1}{4 \cdot 17} = \frac{1}{68} \Rightarrow \frac{x^2}{68} + x + 17 = 0 \Rightarrow x_1 = x_2 = -34.$$

2) $ax^2 + (5 - 3a)x - a = 0$ has two roots of different signs, let $x_1 < 0, x_2 > 0. \Rightarrow$

$$\Rightarrow x_1 x_2 = -\frac{a}{a} = -1 < 0, i.e. when\ a \neq 0\ and\ D = (5 - 3a)^2 + 4a^2 > 0\ both\ roots\ of\ the\ equation\ are\ real$$

and one is positive, the other is negative ($this\ follows\ from\ x_1 \cdot x_2 < 0$).

$$Answer: 1)\ a = 0,\ \ a = \frac{1}{68},\ \ 2)\ a \neq 0.$$