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Problem:

Find all values of the parameter a, for which

1) the equation $4x^2 + 4x = a^2 - 1$ has two different positive roots.
2) the equation $(a-2)x^2 + 2(a-2)x + 2 = 0$ has no roots.

Solution:

1) $4x^2 + 4x = a^2 - 1$ has the roots x_1, x_2, moreover $x_1 \neq x_2, x_1 > 0, x_2 > 0$.

The 1-st method.

$$4x^2 + 4x + 1 - a^2 = 0, \qquad x_1 \neq x_2 \Rightarrow \frac{D}{4} = 4 - 4(1 - a^2) > 0 \Rightarrow a^2 > 0 \Rightarrow a \neq 0.$$

According to Viet's formulas, we have:

$$x_1 + x_2 = -\frac{4}{4} = -1, \text{но } x_1 + x_2 > 0 \Rightarrow we\ got\ a\ contradiction\ , i.e. such\ a\ don't\ exist.$$

The 2-nd method.

$$4x^2 + 4x = a^2 - 1 \Rightarrow (2x+1)^2 = a^2, \Rightarrow x_1 = \frac{a-1}{2}, x_2 = -\frac{a-1}{2}, x_1, x_2 > 0 \Rightarrow \begin{cases} a - 1 > 0 \\ -a - 1 > 0 \end{cases} \Rightarrow -2 > 0.$$

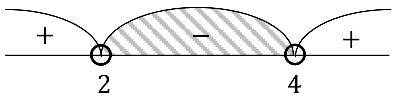
A contradiction.

2) $(a-2)x^2 + 2(a-2)x + 2 = 0$ has no roots.

When $a = 2 \Rightarrow 2 = 0$, contradiction, i.e. has no roots.

$$When\ a \neq 2 \Rightarrow \frac{D}{4} = (a-2)^2 - 2(a-2) = (a-2)(a-4) < 0,$$

$\Rightarrow a \in (2; 4)$, adding $a = 2$ to the interval, we obtain $[2; 4)$.



Answer: 1) $Such\ a\ doesn't exist$, 2) $a \in [2; 4)$.