Problem:

Determine the number of roots of the equation for each value of the parameter a.

$ax^2 + (a+1)^2 x + a + 2 = 0.$

Solution:

Let's find the number of roots of the equation for each parameter a.

The discriminant $D = (a+1)^4 - 4a(a+2) = (a+1)^4 - 4(a+1)^2 + 4 = ((a+1)^2 - 2)^2 \Rightarrow$

$\Rightarrow D = ((a+1)^2 - 2)^2 \geq 0$ for all $a \in \mathbb{R} \Rightarrow$ the equation for each $a \in \mathbb{R}$ has a root. When $D = 0 \Rightarrow$

$\Rightarrow (a+1)^2 - 2 = 0 \Rightarrow a = \pm\sqrt{2} - 1$, in this case the equation has only one root.

When $D > 0 \Rightarrow (a+1)^2 - 2 \neq 0 \Rightarrow a \in \left(-\infty; -\sqrt{2} - 1\right) \cup \left(-\sqrt{2} - 1; \sqrt{2} - 1\right) \cup \left(\sqrt{2} - 1; +\infty\right) \Rightarrow$ the equation has two roots.

Answer: has 1 root when $a = -\sqrt{2} - 1, a = \sqrt{2} - 1$,

has two roots when $a \in \left(-\infty; -\sqrt{2} - 1\right) \cup \left(-\sqrt{2} - 1; \sqrt{2} - 1\right) \cup \left(\sqrt{2} - 1; +\infty\right)$.