Problem:

Find all values of a for which the sum of the real roots' squares of the equation

$x^2 - ax + a - 2 = 0$ is minimal.

Solution:

$x^2 - ax + a - 2 = 0$. Let x_1, x_2 *are the roots of the equation*. If the equation has a real root $\Rightarrow$ the discriminant $D \geq 0$, and it means that the 2-nd root is real too.

According to Viet's formulas:

$$\begin{cases} x_1 + x_2 = a \\ x_1 x_2 = a - 2 \end{cases} \Rightarrow x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = a^2 - 2(a - 2) = (a - 1)^2 + 3 \geq 3,$$

since $(a - 1)^2 \geq 0 \Rightarrow x_1^2 + x_2^2 = 3$ *when* $(a - 1)^2 = 0 \Rightarrow a = 1$ and in this case $D = a^2 - 4(a - 2) = 5 > 0$ $\Rightarrow$ both roots are real.

Answer: $a = 1$, $min\ (x_1^2 + x_2^2)$=3.