



Problem:

Solve for all values of the parameter:

$$a^2(x - 1) - a(2x - 5) = 3x - 6.$$

Solution:

$$a^2(x - 1) - a(2x - 5) = 3x - 6, \quad a^2x - a^2 - 2ax + 5a - 3x + 6 = 0, \quad (a^2 - 2a - 3)x = a^2 - 5a - 6,$$

$$\text{we notice, that } a^2 - 2a - 3 = (a + 1)(a - 3), a^2 - 5a - 6 = (a + 1)(a - 6) \Rightarrow$$

$$\Rightarrow (a + 1)(a - 3)x = (a + 1)(a - 6), (a + 1)((a - 3)x - a + 6) = 0, \Rightarrow \text{if } a + 1 = 0 \Rightarrow a = -1, \text{ any real number is the solution to the equation, i.e. } (-\infty; +\infty).$$

$$\text{If } a + 1 \neq 0 \Rightarrow a \neq -1 \Rightarrow (a - 3)x - a + 6 = 0, (a - 3)x = a - 6, a \neq 3 \Rightarrow \text{the equation has}$$

$$\text{only one solution } x = \frac{a - 6}{a - 3}, \text{ if } a = 3 \Rightarrow \text{since } a - 6 = -3 \neq 0 \Rightarrow \text{the equation has no solution.}$$

Answer: when  $a = -1$ , the solution is  $(-\infty; +\infty)$ ,

$$\text{when } a \neq -1, a \neq 3, \text{ the solution is } x = \frac{a - 6}{a - 3},$$

when  $a = 3$  there are no solutions.