



Problem:

Solve for all values of the parameter.

$$\frac{x-5}{x-1} - \frac{2}{k} = \frac{3}{k(x-1)}.$$

Solution:

$$\frac{x-5}{x-1} - \frac{2}{k} = \frac{3}{k(x-1)} \Rightarrow \frac{x-5}{x-1} - \frac{2}{k} - \frac{3}{k(x-1)} = 0, \quad \frac{(x-5)k - 2(x-1) - 3}{(x-1)k} = 0 \Rightarrow$$

$$\Rightarrow \frac{xk - 5k - 2x + 2 - 3}{k(x-1)} = 0 \Rightarrow \begin{cases} (k-2)x - 5k - 1 = 0 \\ k \neq 0 \\ k \neq 1 \end{cases} \Rightarrow \begin{cases} (k-2)x = 5k + 1 \\ k \neq 0 \\ k \neq 1 \end{cases} \Rightarrow$$

$\Rightarrow$ when $k = 2 \Rightarrow 5k + 1 \neq 0 \Rightarrow 0 \neq 11 \Rightarrow$ there is no solution, when $k \neq 2 \Rightarrow$ the only solution is

$$x = \frac{5k+1}{k-2}, \text{ but } x \neq 1 \Rightarrow \frac{5k+1}{k-2} \neq 1 \Rightarrow k \neq -\frac{3}{4}.$$

$$\text{Answer: } k \in \left\{0; 2; -\frac{3}{4}\right\} \Rightarrow \text{there are no solutions, } k \notin \left\{0; 2; -\frac{3}{4}\right\} \Rightarrow x = \frac{5k+1}{k-2}.$$