Problem:

Find all values of the parameter for which the equation $ax^2 - 4ax + 4a - 5 = 0$ has negative roots.

Solution:

Let's find the values of the parameter a for which the equation has negative roots.

When $a = 0 \Rightarrow -5 \neq 0 \Rightarrow$ there is no solution $\Rightarrow a \neq 0, \Rightarrow$ we have a quadratic equation, by the condition, it has roots $\Rightarrow$

$$\Rightarrow \frac{D}{4} = (2a)^2 - a(4a-5) = 4a^2 - 4a^2 + 5a = 5a \geq 0 \Rightarrow a \geq 0, but\ a \neq 0 \Rightarrow x_{1,2} = \frac{2a \pm \sqrt{5a}}{a} = 2 \pm \frac{\sqrt{5a}}{a} \Rightarrow$$

for the quadratic equation to have negative roots, it's necessary and sufficient, that the smaller root is negative, but evidently

$$x_1 = 2 - \frac{\sqrt{5a}}{a} < 2 + \frac{\sqrt{5a}}{a} = x_2\ (a > 0) \Rightarrow 2 - \frac{\sqrt{5a}}{a} < 0 \Rightarrow \begin{cases} 2 < \frac{\sqrt{5a}}{a} \\ a > 0 \end{cases} \Rightarrow 4 < \frac{5a}{a^2} \Rightarrow 4 < \frac{5}{a}, a > 0 \Rightarrow a < \frac{5}{4} \Rightarrow$$

$$\Rightarrow 0 < a < \frac{5}{4} \Rightarrow a \in \left(0; \frac{5}{4}\right).$$

$Answer$: $\left(0; \frac{5}{4}\right)$.