



Problem:

Solve the irrational equation.

$$\sqrt{3x^2 + 5x + 8} - \sqrt{3x^2 + 5x + 5} = 1.$$

Solution:

$$\sqrt{3x^2 + 5x + 8} - \sqrt{3x^2 + 5x + 5} = 1 \Rightarrow \sqrt{3x^2 + 5x + 8} = 1 + \sqrt{3x^2 + 5x + 5}, \text{ let's square:}$$

$$3x^2 + 5x + 8 = 1 + 2\sqrt{3x^2 + 5x + 5} + 3x^2 + 5x + 5, \Rightarrow \sqrt{3x^2 + 5x + 5} = 3, \text{ let's square again:}$$

$$3x^2 + 5x + 5 = 9 \Rightarrow 3x^2 + 5x - 4 = 0 \Rightarrow x_1 = -\frac{4}{3}, x_2 = 1.$$

Let's check the roots:

$$x_1 = -\frac{4}{3} \Rightarrow \sqrt{16} - \sqrt{9} = 1, \quad x_2 = 1 \Rightarrow \sqrt{16} - \sqrt{9} = 1 \Rightarrow \text{everything's correct.}$$

$$\text{Answer: } x_1 = -\frac{4}{3}, x_2 = 1.$$