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Problem:

Find the domain of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{nx^n}{2^n}.$$

Solution:

Let's use d'Alembert's criterion:

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)x^{n+1}}{2^{n+1}}\frac{2^n}{nx^n}\right| = |x|\frac{1}{2}\frac{1+\frac{1}{n}}{1}\xrightarrow[n\to\infty]{}\frac{1}{2}|x| \Rightarrow$ when $\frac{|x|}{2} < 1$ the series absolutely converges $\Rightarrow x \in (-2; 2)$, when $\frac{|x|}{2} > 1$ the series diverges, at the ends of the segment line we obtain:

$x = -2 \Rightarrow \sum_{n=1}^{\infty}(-1)^n n$, diverges, since the general term of the series does not tend to zero: $n \to +\infty$.

$x = 2 \Rightarrow \sum_{n=1}^{\infty} n$ diverges $(n \to +\infty) \Rightarrow$ the area of convergence of the original series will be $(-2; 2)$.

Answer: $(-2; 2)$.