



Problem:

Find the domain of convergence of the series:

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n!}.$$

Solution:

Let's use d'Alembert's criterion:

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x-1)^{n+1}}{(n+1)!} \frac{n!}{(x-1)^n} \right| = \lim_{n \rightarrow \infty} \frac{|x-1|}{n+1} \xrightarrow{n \rightarrow \infty} 0 < 1 \Rightarrow \text{the series absolutely converges for all}$$

$x \in (-\infty; +\infty), \Rightarrow$ the domain of convergence of the series will be $(-\infty; +\infty)$.

Answer: $(-\infty; +\infty)$.