



Problem:

1. Write the first 5 terms of the series:

$$\sum_{n=0}^{\infty} \frac{n^3}{n+3}$$

2. Write it as a series with a common term:

$$\frac{0}{2} + \frac{1}{3} + \frac{4}{4} + \frac{9}{5} + \frac{16}{6} + \dots$$

Solution:

$$1. \sum_{n=0}^{\infty} \frac{n^3}{n+3} = \frac{0}{0+3} + \frac{1^3}{1+3} + \frac{2^3}{2+3} + \frac{3^3}{3+3} + \frac{4^3}{4+3} + \dots = 0 + \frac{1}{4} + \frac{8}{5} + \frac{27}{6} + \frac{64}{7} + \dots$$

2. Let's notice that the denominators alternate and the numerators are perfect squares.

$$\frac{0}{2} + \frac{1}{3} + \frac{4}{4} + \frac{9}{5} + \frac{16}{6} + \dots = \sum_{n=2}^{\infty} \frac{(n-2)^2}{n}$$