Problem:

Examine the convergence of the series:

$$\sum_{n=1}^{\infty} \frac{n!}{(3n)!}.$$

Solution:

Let's use d'Alembert's test:

We have a positive series,

$$a_n = \frac{n!}{(3n)!} > 0, \quad \Rightarrow \frac{a_{n+1}}{a_n} = \frac{(n+1)!}{\big(3(n+1)\big)!}\frac{(3n)!}{n!} = \frac{n!\,(n+1)}{(3n)!\,(3n+1)(3n+2)(3n+3)}\frac{(3n)!}{n!} =$$

$$= \frac{n+1}{(3n+1)(3n+2)3(n+1)} = \frac{1}{3}\frac{1}{(3n+1)(3n+2)} \underset{n\to\infty}{\longrightarrow} 0 \Rightarrow \lim_{n\to\infty}\frac{a_{n+1}}{a_n} = 0 < 1 \Rightarrow$$

⇒ according to the d'Alembert's test the series converges.

Answer: converges.