Problem:

Examine the convergence of the series:

$$\sum_{n=0}^{\infty} \frac{2n+n^5}{3n^7-7}.$$

Solution:

Let's note that this series is equivalent to the series $\sum_{n=0}^{\infty} \frac{1}{n^2}, \text{ since } \frac{2n+n^5}{3n^7-7} : \frac{1}{n^2} = \frac{(2n+n^5)n^2}{3n^7-7} = \frac{(2n+n^5)n^2}{3n^7-7}$

$$=\frac{1+\frac{2}{n^4}}{3-\frac{7}{n^7}}\underset{n\to\infty}{\longrightarrow}\frac{1}{3}, \text{ but the series }\sum_{n=0}^{\infty}\frac{1}{n^2} \text{ converges} \Rightarrow \text{ the initial series also converges}.$$

Answer: converges.