Problem:

Examine the convergence of the series:

$$\sum_{n=0}^{\infty}\left(\frac{3}{4n+7}\right)^n.$$

Solution:

We have $\frac{3}{4n+7}<\frac{1}{n}, n\geq 1, \Rightarrow \left(\frac{3}{4n+7}\right)^n<\left(\frac{1}{n}\right)^n\leq\frac{1}{n^2}, n\geq 2, \Rightarrow \left(\frac{3}{4n+7}\right)^n\leq\frac{1}{n^2}, n\geq 2,$

but the series $\sum_{n=0}^{\infty}\frac{1}{n^2}<+\infty$ (converges), $\Rightarrow$ the initial series also converges.

$$\left(\sum_{n=0}^{\infty}\left(\frac{3}{4n+7}\right)^n<1+\frac{3}{11}+\sum_{n=0}^{\infty}\frac{1}{n^2}<+\infty\right).$$

Answer: converges.