Problem:

A binary relation is given: $R = \{(x, y) \mid x, y \in \mathbb{N} \wedge\ y \vdots x\}$.

Find: $\delta_R, \rho_R, R^1, R \circ R^{-1}, R \circ R$.

Solution:

$R = \{(x, y) \mid x, y \in \mathbb{N} \wedge y \vdots x\}$.

The domain is $\delta_R = \mathbb{N}$ $(\forall x \in \mathbb{N}, (x, 2x) \in R)$.

The range is $\rho_R = \mathbb{N}$ $(\forall y \in \mathbb{N}, (1, y) \in R)$.

$R^{-1} = \{(x, y) \mid x, y \in \mathbb{N} \wedge (y, x)\ \in R\} = \{(x, y) \mid x, y \in \mathbb{N} \wedge\ x \vdots y\}$.

$R \circ R^{-1} = \{(x, y) \mid \exists z \in \mathbb{N}, (x, z) \in R^{-1} \wedge (z, y) \in R\} = \{(x, y) \mid \exists z \in \mathbb{N}, x \vdots z \wedge y \vdots z\} =$

$= \mathbb{N} \times \mathbb{N}$, since when $z = 1 \Rightarrow x \vdots z, y \vdots z$.

$R \circ R = \{(x, y) \mid \exists z \in \mathbb{N}, (x, z) \in R \wedge (z, y) \in R\} = \{(x, y) \mid \exists z \in \mathbb{N}, z \vdots x \wedge y \vdots z\} \Rightarrow y \vdots x \Rightarrow$

$\Rightarrow R \circ R = \{(x, y) \mid y \vdots x\} = R \Rightarrow R \circ R = R$.