Problem:

Find the general solution of the linear inhomogeneous differential equation with constant coefficients.

$y'' + 2y' - 3y = 5e^{2x}$.

Solution:

This is a linear inhomogeneous equation with constant coefficients ⇒ first let's solve the corresponding homogeneous equation $y'' + 2y' - 3y = 0$. Its characteristic equation will be $\lambda^2 + 2\lambda - 3 = 0 \Rightarrow \lambda_1 = 1, \lambda_2 = -3$ ⇒ the general solution of the homogeneous equation will be:

$y_1 = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} = C_1 e^{x} + C_2 e^{-3x}$.

Now since on the right of the initial equation we have the polynomial of the form $5e^{2x}$, moreover 2 is not the root of the characteristic equation ⇒ we look for the particular solution of the initial equation in the form

$y_2 = ae^{2x} \Rightarrow y_2' = 2ae^{2x}, y_2'' = 4ae^{2x} \Rightarrow y_2'' + 2y_2' - 3y_2 = 5ae^{2x} = 5e^{2x} \Rightarrow a = 1 \Rightarrow y_2 = e^{2x}$

⇒ the desired general solution of the initial equation will be:

$y = y_1 + y_2 \Rightarrow \boxed{y = C_1 e^{x} + C_2 e^{-3x} + \mathrm{e}^{2x}}$.