Problem:

Compare the infinitesimals:

$\alpha(x) = \ln(x^2 + 1)\,, \qquad \beta(x) = x^3 - 2x, \qquad x \to 0.$

Solution:

$\alpha(x) = \ln(x^2 + 1)\,, \qquad \beta(x) = x^3 - 2x, \qquad x \to 0.$

$$\frac{\alpha(x)}{\beta(x)} = \frac{\ln(x^2+1)}{x^3-2x} = \frac{\ln(1+x^2)}{x^2} \cdot \frac{x}{x^2-2} \underset{x\to 0}{\longrightarrow} 0, \qquad \text{since} \quad \frac{\ln(1+x^2)}{x^2} \underset{x\to 0}{\longrightarrow} 1, \qquad \frac{1}{x^2-2} \underset{x\to 0}{\longrightarrow} -\frac{1}{2},$$

$\Rightarrow \alpha(x)$ is an infinitesimal of higher order than $\beta(x)$ $(x \to 0)$.