



Problem:

Find the general solution of the linear homogeneous differential equation:

$$y'' + 4y' = 0.$$

Solution:

$y'' + 4y' = 0$ . This is a second order linear homogeneous equation, let's make a replacement:

$$y'(x) = z(x) \Rightarrow y'' = z' \Rightarrow z' + 4z = 0 \Rightarrow \frac{dz}{dx} = -4z \Rightarrow \frac{dz}{z} = -4dx, \text{ let's integrate } \Rightarrow \int \frac{dz}{z} = -4 \int dx \Rightarrow$$

$$\Rightarrow \ln|z| = -4x + C_0 \Rightarrow |z| = e^{C_0} \cdot e^{-4x} \Rightarrow z = C_1 \cdot e^{-4x}, \text{ where } C_1 \text{ is the arbitrary constant}$$

$$z = y' \Rightarrow \frac{dy}{dx} = C_1 \cdot e^{-4x} \Rightarrow dy = C_1 \cdot e^{-4x} dx, \int dy = C_1 \cdot \int e^{-4x} dx \Rightarrow y = -\frac{C_1}{4} e^{-4x} + C_2,$$

let's denote  $C_3 = -\frac{C_1}{4} \Rightarrow$  we obtain the desired general solution of the initial equation:

$$\boxed{y = C_3 e^{-4x} + C_2} \text{ where } C_2, C_3 \text{ are arbitrary constants.}$$