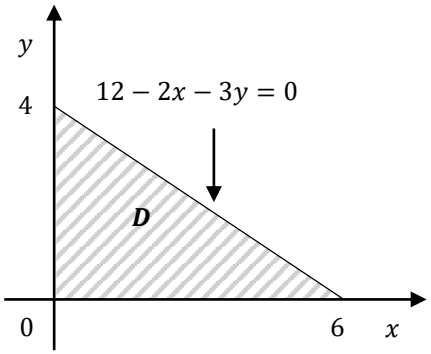
Problem:

Find the double integral over the given area D:

$$\iint\limits_{(D)} (12-2x-3y)\,dxdy, \qquad D:\begin{cases}12-2x-3y=0\\ x=0, \qquad y=0\end{cases}$$

Solution:

Shaded is the area $D, x\in[0;\ 6] \Rightarrow y\in\left[0;\ 4-\frac{2x}{3}\right] \Rightarrow$



$$\Rightarrow \iint\limits_{D} (12-2x-3y)\,dxdy = \int\limits_0^6 dx \int\limits_0^{4-\frac{2x}{3}} (12-2x-3y)\,dy =$$

$$= \int\limits_0^6 \left[\left((12-2x)y-\frac{3}{2}y^2\right)\Bigg|_{y=0}^{y=4-\frac{2x}{3}}\right]dx =$$

$$= \int\limits_0^6 \left((12-2x)\left(4-\frac{2x}{3}\right)-\frac{3}{2}\left(4-\frac{2x}{3}\right)^2\right)dx = \frac{2}{3}\int\limits_0^6 (x-6)^2\,dx = \frac{2}{3}\cdot\frac{1}{3}(x-6)^3\Big|_0^6 = \frac{2}{9}\cdot 6^3 = 48.$$

Answer: 48.