



Условие:

Вычислить несобственный интеграл:

$$\int_{-\infty}^{+\infty} \frac{dx}{x^2 + 2x + 2}.$$

Решение:

$$\begin{aligned}\int_{-\infty}^{+\infty} \frac{dx}{x^2 + 2x + 2} &= \int_{-\infty}^0 \frac{dx}{(x+1)^2 + 1} + \int_0^{+\infty} \frac{dx}{(x+1)^2 + 1} = \lim_{A \rightarrow -\infty} \int_A^0 \frac{dx}{(x+1)^2 + 1} + \lim_{B \rightarrow +\infty} \int_0^B \frac{dx}{(x+1)^2 + 1} = \\ &= \lim_{A \rightarrow -\infty} \tan^{-1}(x+1) \Big|_A^0 + \lim_{B \rightarrow +\infty} \tan^{-1}(x+1) \Big|_0^B = \\ &\quad \text{Graph of } y = \tan^{-1}(x+1) \text{ on the interval } [-\infty, \infty] \text{ is shown below. It is an increasing function passing through } (-1, 0). \\ &\quad \text{At } x \rightarrow -\infty, y \rightarrow -\frac{\pi}{2}. \text{ At } x \rightarrow +\infty, y \rightarrow \frac{\pi}{2}. \\ &\quad \boxed{\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}} \\ &= \tan^{-1} 1 - \lim_{A \rightarrow -\infty} \tan^{-1}(A+1) + \lim_{B \rightarrow +\infty} \tan^{-1}(B+1) - \tan^{-1} 1 = \\ &= \frac{\pi}{2} + \frac{\pi}{2} = \pi, \text{ ибо } \tan^{-1} x \xrightarrow[x \rightarrow +\infty]{} \frac{\pi}{2}, \tan^{-1} x \xrightarrow[x \rightarrow -\infty]{} -\frac{\pi}{2}.\end{aligned}$$

Ответ: π .